RISK AND RETURN: A PARADOX?

Terry A. Marsh
Douglas S. Swanson

WP#1433-83 May 1983

RISK AND RETURN: A PARADOX?

Terry A. Marsh
Douglas S. Swanson

WP#1433-83 May 1983

RISK AND RETURN: A PARADOX?

Terry A. Marsh

Douglas S. Swanson

Sloan School of Management

Massachusetts Institute of Technology

Last Revision: April 1983

First Draft: June 1982

RISK AND RETURN: A PARADOX?

1. Introduction

1.1 Overview

In a recent article in this Review, Professor Bowman has examined the relation between company risk and return within industries, finding that "...in the majority of industries [he] studied, higher average profit [return on equity (ROE)] companies tended to have lower risk, i.e. variance [of ROE] over time." (1980, p.19). He considers this negative correlation between risk and return to be paradoxical relative to much of the business and economics literature. In a later paper, Professor Bowman [1982] has discussed "...explanations [of the negative correlation] involving management and planning factors and...explanations involving firms' attitudes to risk," (p. 33). In contrast, our paper is concerned with the more basic question of whether the negative relation does actually exist, and whether it could indeed be considered paradoxical in light of the extant finance literature.

Clearly, while the type of study undertaken by Professor Bowman could be useful in understanding management behavior, it has important and direct implications at the pragmatic decision-making level. For example, in studies of concentration, barriers to entry, regulatory policy in setting rates. etc., accounting based rates of return like ROE[1] are often used, either because securities market

data for risk and return evaluation are unavailable (e.g., at the divisional level) or because the securities market has already capitalized the economic rents which it is hoped to study cross-sectionally. Thus if a researcher or policy maker doing a cross-section-time series study of barriers to entry knew that, on average cross-sectionally, the (time-series) mean and variance of ROEs were negatively correlated, should this imply that taking risk into account exacerbates cross-sectional differences in time series mean ROEs? Is there any industrial organizational explanation of why barriers to entry per se would induce such a negative relation? Or, if Professor Bowman's result that the negative correlation is primarily an intra-industry phenomenon is correct, should industry regulators adjust rates and hence mean ROE's upward or downward to reflect risk differences?

1.2 Interpretation

Turning to Professor Bowman's results, it is not clear that, in all respects, they should be considered paradoxical at the firm level. If equity is measured as the difference between assets and liabilities which are valued at the historic cost of investment outlays and face value respectively, investors would be delighted to have management undertake projects with high expected rates of return on equity (ROEs) and low risk. Further, if these high ROEs tended to persist through time because of "product cycles" and "harvesting of cash cows" etc., leading to high *ex post* mean ROEs,

it may be that not only would the high mean ROE be associated with a low ex post variance of the ROE, but also with lower uncertainty regarding information about the mean and variance of the ROE.

Nor is the result necessarily paradoxical at a marco level. For example, Fama [1981] finds that changes in the expected real return on capital explains some 40% of the variability in real stock rates of return. He further suggests that real rates of return tend to be negatively correlated with expected inflation because of their common association with changes in industrial production. In sum, high real ROEs seem to be associated with low expected rates of inflation. If, as is commonly believed, variability of inflation is low when expected inflation is low, and a "Fisher effect" holds to any extent at the firm level, high nominal ROEs will tend to occur when the variability of nominal ROEs is low.

1.3 Use of Accounting Numbers

There is an overriding question concerning how strong an interpretation can be made of results based on accounting rates of return. If the value of the firm's assets includes a value for goodwill which approximates the net present value of its projects, the attractiveness of those projects will already be in the equity value, and hence not reflected in the ROE. Or, if a profitable project involves large cash outlays and lagged cash inflows, the firm may have a relatively low ROE over certain intervals of the product cycle and a relatively high ROE at other times even though

much of the risk is resolved during the period of investment (and thus of low ROE). Stated a slightly different way, cross-sectional relations between means and variances of accounting ROEs might be more a statement about accounting techniques than management behavior. The rate of return on equity rather than rate of return on assets will also be subject to variations in the firm's leverage (as will capital market returns on equity).

The use of a ratio of net accounting earnings to book value of assets is itself capable of producing "artificial" results. For example, if accounting depreciation tends to be "undercharged" relative to economic depreciation in times of intense utilization of capacity and high earnings, and vice versa in low earnings periods, then there is an errors-in-variables bias upward in the variance of ROE and, in plausible cases, a bias downward in the mean ROE.

1.4 Strategic and Financial Interpretations

Suppose all the measurement and short run problems with ROE as a measure of firm rate of return are assumed away. Consider two firms A and B. Then, in a world of risk averse investors, if firm A's projects are more risky than firm B's, its hurdle rate of return or cost of capital, ROE*, must be higher. The decision rule for managers centers on what ROE* is appropriate for a given project's riskiness. If firm B happens to have available projects with high ROEs, then it is entirely conceivable that $ROE_A < ROE_B$ even though $ROE_A^* > ROE_B^*$.[2] The implications for "corporate

strategy" would be empty, because the results arise from the simple stipulation that management undertake all positive NPV projects--the real assistance they need is a guide to determining how profitable must a project be to have a positive NPV. To answer this question, it must be possible to dichotomize returns between those demanded as adequate compensation, given investor alternatives, for a project's riskiness--the cost of capital, and the economic rents[3] which measure the extent to which managers beat the alternative. A stronger statement can actually be made about the mean ROE on marginal projects (which equals the cost of capital). If (required) expected returns on these projects (and hence firms) were negatively correlated with the risk (measured according to the appropriate concept), investors could create portfolios by investing directly in the projects with a mean return higher, but risk lower, than that offered by the corresponding securities, which in turn would provide the opportunity for riskless arbitrage.

To interpret the results as bearing on management behavior, it would seem necessary to infer causation from the risk-return results. But the causation could go either way, or even merely illustrate the textbook caveat that correlation might simply represent a common association with a third variable. Suppose management "freezes" the firm's investments. Further suppose that an exogenous, firm-specific, but permanent shock from either the supply or demand side hits the firm, causing future cash flows to become more risky. At the capital market level, the firm's stock

price would fall to generate a higher expected return premium if this risk change is important to investors (i.e., it cannot be diversified away). Thus, at the capital market level, there would be a negative relation between ex post return (reflecting the price drop), and risk in the period of the shock, (all else equal), and a positive relation thereafter. At the accounting ROE level, we might again find that management "signal" their perception of the altered circumstances of the firm by reporting a "low" accounting earnings figure, which would tend to make the ex post average ROE lower while at the same time the "abnormal" reported accounting earning makes the ex post variance of ROE higher. Clearly, increases in firm riskiness are accompanied by changes in expected real cash flows because of the technological structure of the economy.

Continuing to assume away accounting measurement problems in ROE series, the finding of a negative correlation between the mean and variance of ROE would only be paradoxical, relative to all reasonable financial models explicitly or implicitly based on risk aversion, if it arises entirely or partly because high risk firms had low expected ROEs. Since there is nothing unusual about low risk firms having high ROEs, it is the symmetry in Professor Bowman's results that seems counter-intuitive.

Of course, there will be, under certain conditions, an incentive for managers who are maximizing the wealth of the firm's current stockholders to "bet the firm" if its assets become insufficient to pay off fixed obligations. That is, periods of low realized ROEs

might be <u>succeeded by</u> periods of high variance of ROEs. Such "risk seeking by troubled firms" has been studied extensively in the finance literature--e.g., Fama and Miller [1972], Myers [1977], Jensen and Meckling [1976]. However, even if all the questions about the empirical relevance of this "agency problem" could be resolved, (e.g., Fama [1980]), it implies, in the context here, that <u>changes</u> occur in the expected ROEs and variances of ROEs over time.[4] Unfortunately, even with the large sample of ROEs here, it is almost impossible econometrically to adequately estimate such shifting parameters. Hence, we will follow Professor Bowman's study and assume that these parameters are constant.

1.5 Outline

Having (hopefully) given some flavor to what can and cannot be inferred about the risk-return relation from the evidence, we now turn to the evidence itself. In the next section, it is argued that cross-sectional tests of the association between <u>true</u> time series <u>expected</u> ROE's and the true <u>variance</u> of those ROE's is a non-trivial statistical problem. Professor Bowman performs a two-way contingency table test of the association between <u>sample</u> mean ROE's and the <u>sample</u> variance of the ROE's for companies in his sample, but we show that this test is not correct, and may cause an apparent correlation even when no true one exists. In Section 3, we describe our data and the six ROE definitions which we use. As part of our preliminary data analysis, we also repeat Professor Bowman's

categorical data analysis methodology on our sample which contains a much longer time series than Professor Bowman's. Then, in Section 4, we briefly discuss some of the capital markets research bearing on the equilibrium risk-return relation, and how it can lead us to a more specific test of Professor Bowman's hypothesis. Finally, in Section 5, we outline a test for cross-sectional relations between true means and true variance which is developed and applied in Marsh and Newey [1983], and apply it to testing for a relation between means and variances of ROEs. For four different ROE definitions, we find that even after we take out general co-movements in ROEs across companies and industries, there are as many industries displaying positive correlation between ROE means and variances across the companies in the industry as there are the negative correlation reported by Professor Bowman. In the few cases where the correlation, either positive or negative, is significant, it seems to be explained by a single outlier.

2. Tests based on Return on Equity (ROE) Ratios

Professor Bowman's results are based on cross-sectional contingency table tests for association using categorized values of firms' mean returns on equity (ROE) and the variance of those ROE.[2] The ROE means and variances are sorted into High and Low categories, and arrayed in a 2x2 table (or "fourfold table"), which appears as follows:

		ROE Variance		
		High	Low	
Mean ROE	High	n_1	n_2	N_M^H
	Low	n_3	n_4	N_M^L
		N_V^H	N_V^L	N

As used here, the (sample) mean ROE and (sample) variance of ROE are "associated" if higher (or lower) mean ROE occur more frequently among the higher or lower category of variance of ROE. Thus, mean and variance would be considered "unassociated" in Professor Bowman's tests if $n_1/N_V^H = n_2/N_V^L = N_M^H/N$. Note that, as always, association doesn't identify causation in the absence of some economic theory.

We begin by pointing out that this application of categorized data analysis is somewhat different from that typically encountered. It is concerned with the association between two sample moments computed from the <u>same</u> sample of observations, rather than that between two different random variables.

We know from statistical theory that only for finite samples drawn from a <u>normal</u> population,[5] will the sample mean and sample variance be independently distributed (i.e., unassociated). In addition, maximum likelihood estimators of location and scale for asymmetrical distributions are asymptotically uncorrelated (independent) <u>only</u> when they are centered about an origin known as

the "center of location" (Kendall and Stuart [1961, Vol. 2, pp. 64-65]). Arithmetic Brownian motion seems quite impossible as a representation of the stochastic process driving earnings or deflated earnings series, and not surprisingly, studies of the probability distribution of financial statement ratios (e.g., Deakin (1976)) have consistently reported departures from normality in the direction of positive skewness for ratios like ROE. And whilst it might not be appropriate to apply asymptotic results to Professor Bowman's samples (see below), it is worthwhile noting that for both the gamma ($r > 2$) and lognormal distributions, which would be typical of those reported by Deakin, the asymptotic mean and variance estimators are negatively correlated when they are not centered about the appropriate origin under the assumption that the distributions are stationary. In the absence of other problems with the categorical analysis, this means that Professor Bowman's paradox may arise purely from his statistical methodology.[6]

Second, to interpret Professor Bowman's results, his sample size and the independence of his sample observations must be considered. In the first of two samples, his analysis is based on "...each company's average profit and the variability of its profits over the five-year period, 1972 to 1976," (p. 19). In the second, his analyses "...use a nine-year period (1968-1972) for ROE mean and variance rather than a five-year period," (p. 20). Assuming the sampling interval to be annual, this means that the sample moments of each company's ROE are computed using five and nine observations

respectively. Sampling error is not accounted for in the contingency table analysis (it refers to association between "true" mean ROE and "true" variance of ROE). To illustrate the magnitude of the sampling error, we applied the well-known Bienayme-Chebycheff inequality to the first three companies in our sample, described below, over the 5-year period 1972-1976. For these three randomly chosen companies, the mean ROE and its upper and lower 95% confidence intervals, respectively, are (0.177, 0.302, 0.051), (0.051, 0.0187, -0.084), and (0.088, 0.243, -0.066). Of course, these confidence intervals depend upon estimates of the variance of the respective ROEs which themselves will have large standard errors in small samples, and thus should be "bootstrapped", but the point hardly serves to warrant doing that.

The immediately preceding actually assumes observations across companies for each time and observations across time for each company are independent. Neither is true, so things are actually worse than in (1) because, intuitively, the higher the dependence among a given number of observations, the lower the "effective" number of observations. First, ROE's across companies are not independent for, as shown by Ball and Brown [1967], there are market and industry factors influencing firms' accounting earnings. The limitation imposed by the cross-sectional interdependence between ROE's is severe because many statistical methods, <u>including</u> the non-parametric ones used by Professor Bowman, are no longer valid. Second, for any given company, ROEs are not independently

distributed through time. Loosely, the number of "effective" observations on ROEs depends upon the properties of the time series of ROEs. Indeed, if the ROE series is not covariance stationary, its true mean and variance will not even be defined--there is no such thing as _a_ mean and _a_ variance.

Time series models of earnings variables such as earnings per share (EPS) have been studied extensively in the accounting literature, and the evidence is overwhelming that (say) EPS is not covariance stationary for the typical firms, at least in the post-World War II era. Presumably the "runaway growth" of earnings due to inflation, net positive new investment, some deliberate smoothing, etc., help explain this finding. When earnings series are deflated by total assets or equity values, the appropriate generating process for the resulting rate of return series is not as obvious, with somewhat different conclusions being reached in previous studies by Ball and Watts [1972] and Beaver [1970]. Our analysis below tends to support the conclusion that a ratio like the ROE used by Professor Bowman is covariance stationary, and hence, that it is sensible to talk about its mean and variance. However, in one test of mean reversion, Beaver [1970, Table 15] found that it took a period of time like eleven years before high and low ROE firms regressed to a common mean ROE, suggesting that indeed fairly long periods may be required to get reasonably precise estimates of those means and variances.

3. Data and Preliminary Analysis

Professor Bowman's results were based on two different data sets. The first contained 387 companies from 11 industries (two of which were studied for strategic management purposes and nine were randomly chosen from Value Line)[7]. He calculated ROE means and variances for these companies for the five-year interval 1972-1976. Tests of association between the ROE means and variances were based on the 2x2 contingency table analysis described in Section 2. Professor Bowman found that the sample mean ROEs and sample variance ROEs were negatively correlated across the firms in ten industries and positively correlated in the other one. However, he did not perform significance tests for the strength of the negative correlation because "...the low number of companies in each table and the closeness of some of the results to the null hypothesis [of no association] would yield rather weak signals," [1980,p. 20]. Stated differently, the correlation was not statistically significant. When we repeated Professor Bowman's tests on our sample (described below), the association was "significantly negative" for only two industries out of thirteen (though we argued above that such tests are flawed). Professor Bowman seems to judge significance by a binomial-type test, reasoning under the null hypothesis, half of the correlation should be negative, and half should be positive (ignoring zero correlation), whereas ten of the eleven industries he studied display negative association.

In his second sample, Professor Bowman studies 1572 firms in 85 industries covered by Value Line over the nine-year interval of 1968 to 1976. To measure the correlation between ROE mean and ROE variance he again formed 2x2 contingency tables of these sample estimates. The sum of the values in the low/high and high/low quadrants of the table were divided by the sum of the values in the high/high and low/low quadrants. A quotient greater than one would indicate negative correlation. He found 56 industries with negative correlation, 21 with positive correlation, and 8 with no correlation. He implies that these results again represent significant negative intra-industry correlation (using a binomial test) between ROE mean and ROE variance.

Finally, using the first sample, Professor Bowman formed one contingency table using all 295 firms from the 9 Value Line industries. This contingency table indicated slight negative correlation, but not at any significant level. Also, using the second sample, he formed a contingency table based upon rank orders of industry ROE mean and ROE variances. Finding insignificant negative correlation, he concluded that there is negative correlation between ROE mean and ROE variance within an industry, but not on an aggregate basis.

We have not recreated Professor Bowman's sample. We have constructed a substantially longer time series of ROEs starting from an original sample of earnings per share for 175 firms supplied to us by Ross Watts at the University of Rochester. That sample

includes a randomly selected 50% of the firms which meet the following criteria:

1. All the firm's quarterly and annual earnings per share numbers are available in the <u>Wall Street Journal Index</u> (WSJI) over the period January 1958 through December 1969.

2. The firm's share price relatives are available on the Wells Fargo Bank daily file for the period July 2, 1962 through July 11, 1969.

3. The firm's shares are listed on the New York Stock Exchange for the period July 1, 1962 through September 27, 1968.

4. The firm's fiscal year is constant in the period January 1958 through December 1969.

All of the earnings-per-share (EPS) numbers obtained from the WSJI are adjusted for stock splits and stock dividends. From this data set we have used 135 firms. The 40 remaining firms were eliminated from our sample because they didn't have fiscal years ending in March, June, September, or December, (since we were interested in analyzing the data on a quarterly calendar), or because they were not on the 1978 COMPUSTAT Industrial Quarterly File. Earnings data from 1970 to 1981 was obtained from the vector of Primary Earnings Excluding Extraordinary Items on the COMPUSTAT Quarterly File. These earnings were also adjusted for splits and stock dividends back to January 1, 1958.

ROE was calculated by deflating these earnings series by several different bases:

1. By Book Value Per Share of Equity at year end (adjusted for splits and stock dividends)[7] defined as: book value of common equity, divided by shares used for calculating Primary EPS). Since these items are available only on the COMPUSTAT Industrial Annual File, this ROE series was calculated only on an annual basis. Also, since the book values were not available for most firms until 1963, the series extends only from 1963 to 1981.

2. By Book Value Per Share of Equity at the beginning of the year. This series is similar to (1) and covers 1964-1981.

3. By Market Value Per Share of Equity at the end of the year (adjusted for splits and stock dividends) as obtained from the CRSP (Center for Research in Security Prices) Monthly Stock Returns (MSR) file from 1963 to 1981. This series was primarily constructed for comparison with the results from (1) and (2).

4. By end of quarter Market Value per share of Equity (adjusted for splits and stock dividends) obtained from the CRSP monthly stock return file and the Wall Street Journal (as a few companies were not listed on the NYSE and thus not on the MSR file at the beginning of our time period). This sample spanned 1958 I to 1981 IV.[8]

5. By compounding the quarterly ROE in (4) to form annual ROE figures. This series covered 1958 to 1981.

6. By the Market Value per share of Equity (adjusted for splits and stock dividends) measured three months after the end of the quarter to which earnings pertain. This series is similar to (4) and (5) and covers 1958 I to 1981 III.

By nature of the ROE ratio, all three equity valuation measures are meant to standardize firms' earnings for scale or size of the firm. There is some reason to believe (6) will induce less "noise" into the ROE series than (3), (4), or (5). The reasoning is as follows: Ball and Brown [1968] show that when a firm's announced earnings differ from those expected, the market value of its stock reacts in the same direction. Further, of all the information about an individual firm which becomes available during a year, about one-half or more is captured in that year's income number, and that, of the information in the income number, some 85%-90% is impounded in the stock's price by the month of its announcement. In the month of the earning's announcement, surprises in that announcement contribute about 20% of the value of all information. Thus, scaling earnings by a post-announcement market value of equity adjusts for the deviation of EPS from that expected. It will reduce the variability of the ROE caused by "noise" and hence increase the precision of estimation of the mean and variance of the ROE. Since the hypotheses relate to the cross-sectional relation between mean and variance of "permanent" ROEs, elimination of the transitory element should enhance the power of our tests.

If the logic of our method (6) is to be preserved, we must ensure that if market values are measured three months after the end of the quarter earnings announcements are made by then. We have not collected announcements for all the firms in our sample, but rely upon a sample of quarterly earnings announcements for 577 firms over

the period 4/75-3/77 collected by Reinganum [1981].[9] He found the distribution of quarterly earnings announcements to be:

Quarter	Month of Release of Quarterly Earnings +1	+2	+3
4/75	124	325	105
1/76	455	109	1
2/76	453	103	4
3/76	424	128	2
4/76	124	316	106
1/77	441	105	3
2/77	427	117	1
3/77	426	105	4

The +1, +2, and +3 months are the first, second, and third months following the fiscal quarter close, respectively. The +1, +2, and +3 columns contain the number of firms in the sample that publicly released their quarterly earnings during that month.

Since there is every reason to expect Reinganum's results to apply to the companies in our sample, we feel confident that our post-announcement prices are capturing most of the surprises in EPS numbers.

Sample autocorrelations were calculated for the ROE series (4), which was defined to be EPS/End-of-quarter market value of equity, for the time periods 1958-1969, 1970-1981, and 1958-1981, for each of the 135 firms in our sample. The autocorrelations for 1, 2, 3, and 4 quarter lags were computed. Auto- correlations at a one-year lag were computed for the annual EPS/End-of-year market value of equity series (3), EPS/End-of-year book value of equity series (1), and the EPS/Beginning-of-year book value of equity series (2). The mean and median of these autocorrelations for the 135 firms are

reported in Table 1. Means and medians of the ROE autocorrelations for series (6) were similar to those for series (4) and (5). Our mean for the autocorrelation at a one-year lag of the EPS/End-of-year market-value-of equity series (3) over the period 1958-1981 of 0.632 is above that reported by Beaver [1970] who examined a sample of 57 "industrial" NYSE- listed firms with data on the Compustat over the period 1949-1968 and obtained an average point estimate of first-order serial autocorrelation of 0.40.[10] Beaver's figure of 0.48 for the EPS/Beginning-of-year book value of equity over the 1949-1968 subperiod is, however, quite close to our figure of 0.52 for the similar series (2) and time period 1964-1981.

Histograms showing the distribution of lag one one-quarter and one-year autocorrelations of EPS/End-of-period market value of equity for the period 1958-1981 are given in Figures 1 and 2 respectively. A histogram of the lag one one-year autocorrelation of the EPS/Beginning-of-year book value of equity for the period 1964-1981 is shown in Figure 3. It can be seen that the 1958-1981 EPS/End-of-year market value of equity series displays much larger autocorrelation over the 1958-1981 period than in either of the subintervals 1958-1969 or 1970-1981. This is because the realized average ROE by this definition was much larger in the 1970s than in the 1960s, so that ROEs in the 1970s tend to be above, and the ROEs in the 1960s below, the overall period mean.[11] This apparent trend in ex post ROEs explains why our lag 1 sample autocorrelation figure for the annual series EPS/End-of-year market value of equity

is higher than the same lag 1 autocorrelation for EPS/Beginning-of-year book value of equity, which is the opposite of Beaver's result.

With our larger sample, we decided to replicate Professor Bowman's tests as part of our preliminary data analysis, although we argued above that his procedures could produce incorrect conclusions. Sample mean and sample variances of ROE were calculated, for each firm, on a quarterly and annual basis for the EPS/End-of-period market value of equity series (3)-(6) over the intervals 1958-1969, 1970-1981, and 1968-1981, and on an annual basis for the EPS/End-of-year book value of equity series (1) for 1963-1981 and the EPS/Beginning-of-year book value of equity series (2) from 1964-1981. Firms were sorted based on their SIC 2 digit industry code as obtained from the 1978 Annual Industrial COMPUSTAT tape dated 9-21-78, (1978 was the last year all companies used here were listed). Only industries containing at least four firms of the 135 firms were included in our analysis (a total of 13 industries).[12] A list of these industries and the number of firms used is shown in Table 2. Six firms changed their SIC classification between 1978 and 1981 and 12 firms were dropped from the tape between 1978 and 1981 because of takeover and merger.

Two-way contingency tables like those used in Bowman [1980] to show the association between high and low categories of mean ROEs and variances of ROEs were constructed for each industry and for the sample as a whole for the six different ROE definitions

(1)-(6).[13] Results for the annual EPS/End-of-year market value of equity from 1958-1981 and annual EPS/Beginning-of-year book value series from 1964-1981 are shown in Table 3 and Table 4, respectively.

For the annual EPS/end-of-year market value of equity series from 1958-1981, nine industry contingency tables indicated positive correlation, three showed negative correlation and one had no correlation. However, only one industry, Chemicals and Allied Products, had a significant (positive) correlation at the 95% confidence level when a chi square test was used. A contingency table of the whole sample showed positive corelation at the 99% confidence level. Spearman tests indicated nine industries had positive correlation and four had negative correlation.

For the annual EPS/Beginning-of-year book value series from 1964-1981, four industry contingency tables indicated positive correlation, five negative correlation, and one showed no correlation. Chi-quare tests were run and none of these results were significant at the 95% confidence level. A contingency table of the whole sample showed slightly negative, but insignificant, correlation. Spearman tests indicated five industries with positive correlation and seven with negative correlation.

Table 5 summarizes the results of the other series, which are not significantly different.[14] It tabulates the number of industries with positive and negative correlation between sample ROE means and variances based upon 2x2 contingency tables and Spearman tests. As just mentioned, the only industry having any significant

correlation in any of the series is that with SIC code 28 (Chemicals and Allied Products), and it has positive correlation. All series, except the two formed by dividing EPS by book value, had more industries with positive correlation than negative correlation, based upon the 2x2 contingency tables and the Spearman tests.

Table 6 presents the results of 2x2 contingency tables and Spearman tests on the entire cross-sectional sample of estimated ROE means and variances for each of the ROE series. Both tests indicate positive correlation for all series adjusted by market values (Series (3)-(6)). The chi square statistics for these series indicate all of the 2x2 contingency tables are significant at the 99% confidence level. Both of the ROE series adjusted by book values are negatively correlated, but not significantly. These results could indicate that part of Professor Bowman's observations are due to his definition of ROE.

4. Risk and Return for Firm's Versus Risk and Return for Securities

The relation between risk and expected rates of return on common stocks has been studied extensively, and so it is convenient to begin our discussion "at the investor level." To a stockholder, returns are made up of dividends and capital gains. Under robust assumptions with respect to the multivariate distribution of stock returns, the following linear regression function or "market model" holds:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}, \quad L = 1,\ldots,N; \quad t = 1,\ldots,T$$

(1)

where R_{it} is the rate of return on security i, R_{Mt} is the average rate of return on all stocks, and ε_{it} is a disturbance with

$E(\varepsilon_{it}) = E(\varepsilon_{it}|R_{Mt}) = 0$. The traditional one period capital asset pricing model implies a constraint across the coefficients $\{\alpha_i,\beta_i\}$.

It is easy to see what (1) says about the mean and variance of security i's rate of return unconditional on R_{Mt}:

$$E(R_{it}) = \alpha_i + \beta_i E(R_{Mt}) \qquad (1)$$
$$\sigma^2(R_i) = \beta_i^2\sigma^2(R_M) + \sigma^2(e_i) \quad .$$

(2)

That is, given the mean $E(R_{Mt})$ and variance $\sigma^2(R_{Mt})$ of the market rate of return, along with the variance of return of security i attributable to non-market factors, $\sigma^2(\varepsilon_i)$, (1) and (2) suggest that $E(R_i)$ and $\sigma^2(R_i)$ will be

related as long as $E(R_{mt})$ is related to $\sigma^2(R_m)$. However, if $\beta_i > 0$ (it is unity on average across all assets), $E(R_i)$ and $\sigma^2(R_i)$ will be positively related,[15] which means that the common contribution of the market factor to both security means and variances would induce a positive relation which is in the opposite direction to that reported by Professor Bowman.

Because of the common market factor which we know, ex ante, will induce a relation between security return means and variances, interest also centers on the relation between a security's mean rate of return and that proportion of its variance "left over" after the market factor is taken out, which is $\sigma^2(\varepsilon_i)$ in (2). There is some evidence that $\sigma^2(\varepsilon_i)$ estimates are positively correlated with estimates of β_i across NYSE stocks, which in turn would again imply a positive relation between $E(R_i)$ and $\sigma^2(\varepsilon_i)$.

Turning now to the firm (as opposed to investor) level, it is necessary to make the transition from the rate of return on the firm's stock to the firm's ROE as defined earlier and as used in the Bowman study. In the simplest case of a firm with only common stock outstanding, which paid out all its net cash inflows each year as dividends, was on a cash basis for accounting, and had its assets valued at market (including the market value of goodwill or the net present value of the firm), the stock rate of return would be equal to ROE as defined here.[16] However, typical accrual accounting earnings (and assets) figures will only measure with error a firm's

net cash inflows, and typical asset valuation techniques do not measure assets (or goodwill if it appears) "at market," and so $E(ROE_{it})$ need not equal $E(R_{it})$. Nevertheless, analogs of (1) based on ROE_{it} rather than R_{it} have been investigated and found to work "reasonably well." Beaver, Kettler and Scholes [1970] studied the degree of association between several accounting measures of risk and market model beta estimates computed from stock returns. They found that variability of an earnings/price ratio similar to definition (3) of the ROE here has as high a contemporaneous correlation with market betas[17] as accounting earnings betas for 307 Compustat firms which they studied over the period 1947-1965.[18] In addition, accounting risk measures gave as good one-step ahead predictions of market betas as the past series of market betas. Likewise, Ball and Brown [1967] found that 35%-40% of the variability in a firm's annual earnings numbers can be associated with the variability of earnings numbers averaged over all firms, i.e., "the market" earnings.

Hence, we conclude that one-period models in finance suggest two different ways to interpret "an association" across firms between the (time series) mean and variance of their ROEs:

(1) Between the mean ROE and "total" variance of the ROE.

(2) Between the mean ROE conditional on the earnings outcomes for all

firms in general and the variance of the ROEs not explained by systematic correlations in ROEs across all firms.

Since only (2) is inconsistent with current one-period asset pricing models in finance, and since (1) actually contains a built-in bias against finding the result reported by Professor Bowman if all his firms had marginal mean ROEs equal to the cost of capital, we focus on tests of (2) in the following section.

5. Test Procedure and Results

Sample observations consist of the multivariate time series ROE_{it}, $i = 1,\ldots N$, $t = 1,\ldots,T$. The test procedure consists of making two transformations of these series and then applying a parametric error components procedure which is developed and applied in Marsh and Newey [1983] to test for cross-sectional association between time series means and time series variances.

The first transformation is designed to eliminate autocorrelation in a time specific component of the $\{ROE_{it}\}_t$.[19] It is shown in Appendix A that this transformation does not affect the hypothesis test which we wish to perform. The variable $\{ROE_{it}\}$ will hereafter be defined as the transformed variable given in Appendix A.

Second, as discussed in the previous section, we are interested only in the cross-sectional association between (time-series) mean ROEs and the (time series) variances of these ROEs which is not due to common cross-sectional association between the ROEs predicted by extant asset pricing models. To eliminate the common "factor," the ROE_{it}'s are again redefined as the residual ROE_{it} in the following model:

$$ROE'_{it} = b_i ROM_t + c_i ROI_{it} + ROE_{it} \tag{3}$$

where ROE'_{it} is the ROE variable as transformed in Appendix A, ROM_t is the equally-weighted average ROE for all 135 companies in our sample in period t, ROI_t is the average industry ROE in period t, and ROE_{it} is the final rate of return variable for which we

wish to test cross-sectional association between mean and variance.

With the above two transformations, the test procedure fits within an error-components model as follows:

$$\tilde{ROE}_{it} = \alpha_i + \tilde{\xi}_i + \tilde{\nu}_{it} \tag{4}$$

where: α_i is the "grand mean" of ROEs across all time periods and all firms, $\tilde{\mu}_i$ is a random firm-specific, but time-invariant, component of the mean with $E(\tilde{\mu}_i) = 0$, and $\tilde{\eta}_{it}$ is the remaining time-varying but firm-specific disturbance with

$E[\eta_{it}|\mu_i] = 0, \quad E[\eta_{it} \cdot \eta_{i\tau}|\mu_i] = 0,$

$\tau \neq t.$

In (4), the test for cross-sectional association between the time series means and variances of ROE_i, $i = 1,\ldots,N$ is a test of whether:

$$H_0: \quad E[\eta_{it}^2|\mu_i] = \sigma_{\eta i}^2 \tag{5}$$

The alternative hypothesis which we consider here[20] is

$$H_A = E[\eta_{it}^2|\mu_i] = g(\mu_i) \tag{6}$$

One of the advantages of the test here (like that of the heteroscedasticity tests of Breusch and Pagan [1979], Engle [1982], and White [1980] with which it shares similarities), is that the form of $g(\cdot)$ need not be specified a priori.

A brief outline of the test is given in Appendix B. (It is explained more fully in Marsh and Newey [1983].) The test statistic is:

$$S = \frac{q_N^2}{\hat{V}(q_N)} \quad . \tag{7}$$

where:

$$q_N = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \bar{r}_{i.} \hat{\sigma}^2_{i.} - \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \bar{r}_{i.} \right) \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \hat{\sigma}^2_{i.} \right) - \frac{1}{T} \left[\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \hat{\mu}_{3i.} \right] \tag{8}$$

$$\hat{V}(q_N) = \left[1, \quad -\frac{1}{N} \sum_{i=1}^{N} \hat{\sigma}^2_{i.}, \quad -\frac{1}{N} \sum_{i=1}^{N} \bar{r}_{i.}, \quad -\frac{1}{T} \right] \hat{\Omega} \begin{bmatrix} 1 \\ -\frac{1}{N} \sum_{i=1}^{N} \hat{\sigma}^2_{i.} \\ -\frac{1}{N} \sum_{i=1}^{N} \bar{r}_{i.} \\ -\frac{1}{T} \end{bmatrix} \tag{9}$$

$$\hat{\Omega} = \frac{1}{N-1} \{ \sum_{i=1}^{N} \begin{bmatrix} (\bar{r}_{i.}\hat{\sigma}^2_{i.})^2 & \bar{r}^2_{i.}\hat{\sigma}^2_{i.} & \bar{r}_{i.}(\hat{\sigma}^2_{i.})^2 & \bar{r}_{i.}\hat{\mu}_{3i.}\hat{\sigma}^2_{i.} \\ & \bar{r}^2_{i.} & \bar{r}_{i.}\hat{\sigma}^2_{i.} & \bar{r}_{i.}\hat{\mu}_{3i.} \\ & & (\hat{\sigma}^2_{i.})^2 & \hat{\sigma}^2_{i.}\hat{\mu}_{3i.} \\ & & & (\hat{\mu}_{3i.})^2 \end{bmatrix}$$

$$- \frac{1}{N} \begin{bmatrix} \sum_{i=1}^{N} \bar{r}_{i.}\hat{\sigma}^2_{i.} \\ \sum_{i=1}^{N} \bar{r}_{i.} \\ \sum_{i=1}^{N} \hat{\sigma}^2_{i.} \\ \sum_{i=1}^{N} \hat{\mu}_{3i.} \end{bmatrix} [\sum_{i=1}^{N} \bar{r}_{i.}\hat{\sigma}^2_{i.}, \sum_{i=1}^{N} \bar{r}_{i.}, \sum_{i=1}^{N} \hat{\sigma}^2_{i.}, \sum_{i=1}^{N} \hat{\mu}_{3i.}] \} \tag{10}$$

where:

$$r_{it} \equiv ROE_{it}$$

$$\bar{r}_{i.} = \frac{1}{T} \sum_{t=1}^{T} r_{it}, \; \bar{\nu}_{i.} = \frac{1}{T} \sum_{t=1}^{T} \nu_{it}$$

$$\hat{\sigma}^2_{i.} = \frac{1}{T-1} \sum_{t=1}^{T} (r_{it} - \bar{r}_{i.})^2 = \frac{1}{T-1} \sum_{t=1}^{T} (\eta_{it} - \bar{\eta}_i)^2$$

$$\hat{\mu}_{3i.} = \frac{T}{(T-1)(T-2)} \sum_{t=1}^{T} (r_{it} - \bar{r}_{i.})^3$$

The test statistic S in (7) is asymptotically (in N) distributed as $\chi^2(1)$.

The test was run on series (1), (2), (4), and (6). Test statistics and direction of correlation for each industry and the whole sample are listed in Table 7. For both Series (1) and (2), industry 35 (Machinery, Except Electrical) has positive correlation at the 95% confidence level. Industry 22 (Textile Mill Products) has significant positive correlation for series (1), but not for series (2). (In fact, it is slightly negative in that sample.) The small size of this industry (6 firms) may be the cause of this result. No other significant positive or negative correlation is evident in these two series.

For series (4), industries 22 (negative correlation, $\chi^2(1) = 3.76$), 28 (positive correlation, $\chi^2(1) = 3.12$), and 32 (negative correlation, $\chi^2(1) = 4.46$) have "fairly significant" levels of correlation. However, these samples are quite small (6, 11, and 7 firms, respectively) and it appears that a single firm within each of the industries may dominate the results, especially for industries 22 and 32. If just a single firm outlier from each of these industries is omitted, the direction of the correlation does not change, but now the test statistics are 1.52, 2.59, and 2.06, respectively, which are insignificant. For series (6), no industries show any significant levels of correlation.

The whole sample of 135 companies displays positive, although

insignificant, correlation for all 4 of the series. For the 4 series, the number of industries within a series with positive correlation is 8, 6, 7, and 7, respectively, and the number with negative correlation is 5,7,6, and 6, respectively. This does not indicate any type of intra-industry correlation between ROE mean and variance, either negatively or positively.

Except for the positive correlation in industry 35 for series (1) and (2) and the positive correlation for all of the 4 whole sample series, we find no significant correlation between average ROEs and their variance. Thus, the results seem contrary to Professor Bowman's.

6. Summary

Professor Bowman's work has made a valuable contribution in terms of the interest it has focused on the relation between risk and return. What we hope to have shown is that estimation of this relation needs to be done carefully. When it is, we find that the paradoxical relation reported by Professor Bowman disappears. Our conclusions from this would appear to depend more on the definition and source of interest in accounting rates of return which differ from capital market returns, for otherwise there already exists a good deal of theory and evidence on the association between risk and return.

FOOTNOTES

1 A rate of return on total assets rather than on equity should be used in such cases because it abstracts from capital structure.

2 Assuming projects are continuous, both firms A and B will continue to save and invest funds until the point at which their marginal ROEs equal their respective ROE*s. Firm B's average ROE can exceed firm A's average ROE in spite of its lower average risk level and hence lower ROE_B^*.

3 In small samples, "economic profits" will also be an important component of measured ex post ROEs, but we defer discussions of sample size to later.

4 Black [1976] suggests that, at least for capital market data, this is a distinct possibility. We should note, however, that Holland Myers [1980, p. 323] found that "... the most interesting feature of the series [of ratios of operating income to market value averaged across all manufacturing and nonfinancial corporations, which is the unlevered equivalent of one of our ROE series used below] was its stability, at least from the mid-1950s through 1976."

5 Professor Bowman also reports that "...more powerful nonparametric procedures of rank orders and Spearman tests have been used in a study which replicated and substantiated [his] findings," (p. 20). Incidentally, we have not been able to confirm the assertion that rank order methods are, or in this application uniformly, more powerful than alternative tests of association. Brown and Benedetti [1977] investigated estimators of asymptotic standard errors (and thus test statistics) for several tests of association in 4x4 and 8x8 tables with sample sizes of 25, 50, and 100. They found in simulation that critical regions for the correlation and rank correlation tests do not always completely overlap those of the other test statistics, but there is no indication that the rank correlation should be preferred in the sense that its critical region is "closer" to the true size in finite samples. So far, we have not found any complete comparison of the power curves (i.e., relative rejection rates over the alternative hypothesis space). For readers interested in this power question, Hutchinson [1979] provides an extensive set of references.

6 The proposition turns out to be an if and only if proposition, i.e., if the mean and variance are independent, the population must be normal (Kendall and Stuart [1977, Vol. 1, Exercise 11.19]).

7 It is the normality of the sample ROEs which is relevant to independence of the sample mean and variance. We note further it was assured in one of the early approaches to contingency table analysis (Pearson [1900]) that the variables in the table (here the sample mean and sample variance were themselves drawn from a discretized bivariate normal distribution. The analysis has more recently been applied under more general conditions (Goodman [1981]).

8 Figures to adjust prices and earnings were obtained from the Monthly Stock Master file (MSM) and from Moody's for the firms not on the NYSE. End of period stock market prices were used because some firms had rights issues which may be viewed as a mixture of stock split and new issue. Since rights issues were included as splits on the MSM file all earnings and prices were adjusted similarly.

9 ROE series (4) and (5) were also divided into subintervals of 1958-1969 and 1970-1981 because of the difference in the sources of earnings series (some of the Watts earnings may contain extraordinary items) obtained from WSJ index and Moody's.

10 Only 535 firms survived until the end of his sample period.

11 Ball and Watts [1972] concluded that, on average across firms, a similar ratio had even higher autocorrelation than ours, and could be regarded as close to a martingale. Foster [1978, Table 4.9] computed the ROEs for 733 firms with data available on the Compustat tape for 1957-1975, and found the average first and second order serial correlation coefficient estimates to be 0.496 and 0.195 respectively.

12 Of course, this just says that the autocorrelation estimates which apply a constant mean will be incorrect if the constant mean assumption is incorrect.

13 The χ^2 test given in Section 5 is asymptotic, so the minimum of 4 firms may not be strict enough. In the Tables, the number of firms in each industry is given so that the reader can assess to his or her own satisfaction the adequacy of the asymptotic assumptions. We note that the usual methods of contingency table analysis used by Professor Bowman (and ourselves in the preliminary data analysis) can give very different results when the cross-sectional sample of firms is small (e.g., Cox and Plackett [1980], Mantel and Hankey [1975], Hutchinson [1979]).

14 For those cases with an odd number of cross-sectional observations, the median (across firms) ROE mean and ROE variance were counted as half in the low and half in the high parts of the table.

15 These other results are available from the authors upon request.

16 For the typical security, about 20%-30% of its variance will be attributable to the common market factor.

17 It is in this pure case that Professor Bowman's statement that the anomaly he reports "...at the level of the firm...can be eliminated in the stockholder markets by the pricing of securities", (p. 25) makes sense.

18 Point estimates in the 0.4-0.6 range.

19 As Beaver, Kettler and Scholes emphasize, with only nine (annual) observations, there is some difficulty in dichotomizing variance of ROE into its systematic and nonsystematic components.

20 This transformation does not (nor is it intended to), eliminate all the autocorrelation in $\{ROE_{it}\}$, $i = 1,...,N$, since the firm-specific, but time-invariant, component μ_i in (5) below induces such autocorrelation.

21 If H_o were rejected, we would be intersted in more specific subsets of this alternative parameter space, e.g., Professor Bowman's results would suggest $g' < 0$.

REFERENCES

Amemiya, T., 1973, "Regression Analysis when the Variance of the Dependent Variable is Proportional to the Square of Its Expectation," Journal of the American Statistical Association, (June), 68 (344), Theory and Methods Section, 928-934.

Ball, R. and P. Brown, 1967, "Some Preliminary Findings on the Association Between the Earnings of a Firm, Its Industry, and the Economy," Supplement to the Journal of Accounting Research, Empirical Research in Accounting: Selected Studies, 55-77.

Ball, R. and P. Brown, 1968, "An Empirical Evaluation of Accounting Income Numbers," Journal of Accounting Research, (Autumn), 6(2), 610-629.

Ball, R. and Watts, R., 1973, "Some Time Series Properties of Accounting Numbers." The Journal of Finance, XXVII(3), 663-681.

Beaver, W., P. Kettler, and M. Scholes, 1970, "The Association Between Market Determined and Accounting Determined Risk Measures," The Accounting Review, XLV(4), (October 1970), 654-682.

Black, F., 1976, "Studies of Stock Price Volatility Changes," Proceedings of the American Statistical Association, Business and Economics Statistics Section, 177-181.

Bowman, E.H., 1980, "A Risk/Return Paradox for Strategic Management," Sloan Management Review, (Spring), 17-31.

Bowman, E.H., 1982, "Risk Seeking by Troubled Firms," Sloan Management Review, (Summer), 23(4), 33-42.

Breuseh, T.S., and H.R. Pagan, "A Simple Test for Heteroscedasticity and Random Coefficient Variation," Econometrica, 47(5), (September 1979), 1287-1294.

Brown, M.B., and Benedetti, J.K., 1977, "Sampling Behavior of Tests for Correlation in Two-Way Contingency Tables," Journal of the American Statistical Association, (June), 72 (358). Applications Section, 309-315.

Cox, M.A.A., and R.L. Plackett, 1980, "Small samples in contingency tables," Biometrika, 67 (1), 1-13.

Deakin, E.B., 1976, "Distributions of Financial Accounting Ratios: Some Empirical Evidence." Accounting Review, 90-96.

Engle, R.F., 1982, "Autoregressive Conditional Heteroscedasticity With Estimates of the Variance of United Kingdom Inflation," Econometrica, 50(4), (June), 987-1007.

Fama, E.F., 1980, "Agency Problems and the Theory of the Firm," Journal of Political Economy, 88 (2), April, 288-307.

_____, and M.H. Miller, 1972, The Theory of Finance, New York: Holt, Rinehart and Winston.

Foster, G., 1978, Financial Statement Analysis, Prentice-Hall, Inc., Englewood Cliffs, NJ.

Goodman, L.A., 1981, "Association Models and the Bivariate Normal for Contingency Tables with Ordered Categories," Biometrika 68 (2), 347-355.

Holland, D.M., and Myers, S.C., 1980, "Profitability and Capital Costs for Manufacturing Corporations and All Nonfinancial Corporations," American Economic Review, 70(2), 320-325.

Hutchinson, T.P., 1979, "The Validity of the Chi-squared Test when Expected Frequencies are Small: A List of Recent Research References," Communications in Statistics: Theory and Methods, A8 (4), 327-335.

Jensen, M.C., and W.H. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure, Journal of Financial Economics (3) October, 305-360.

Kendall, M., and A. Stuart, 1977, The Advanced Theory of Statistics. Volume 1, 4th Ed.; New York: Macmillan Publishing Company, Inc.

Mantel, N., and B.F. Hankey, 1975, "The Odds Ratio of a 2x2 Contingency Table," The American Statistician. 29 (4), November, 143-145.

Marsh, T.A., and Newey, W.K., 1983, "Tests of Cross-sectional Association between Means and Variances: Methodology and Applications to Stock Returns and International Inflation Rates," Unpublished working paper, M.I.T.

Myers, S.C., 1977, "Determinants of Corporate Borrowing," Journal of Financial Economics, 5 (2) November, 147-175.

Pearson, K., 1900, "Mathematical Contribution to the Theory of Evolution VII: On the Correlation of Characters not Quantitatively Measurable," Phil. Trans. R. Society. London, 195A, 1-47.

Reinganum, M.R., 1981, "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings' Yields and Market Values," Journal of Financial Economics, 9, 19-46.

White, H., 1980, "A Heteroscedasticity-Consistent Covariance Matrix and a Direct Test for Heteroscedasticity," Econometrica, 48 (4) May, 817-838.

Zellner, A., "Causality and Econometrics," in K. Brunner and A.H. Meltzer, eds., Three Aspects of Policy and Policymaking, Carnegie-Rochester Conference Series, Vol. 10, North-Holland, 9-54.

Appendix A

In Appendix B, the rate of return on firm i in period t, ROE^*_{it}, is transformed to ROE_{it}, where the ROE_{it} satisfy:

$$ROE_{it} = \alpha + \mu_i + \eta_{it} \tag{A.1}$$

$$E(\eta_{it},\eta_{i\tau}) = \begin{cases} \sigma^2_\eta & t = \tau \\ 0 & t \neq \tau \end{cases} \tag{A.2}$$

Suppose
$$ROE^*_{it} = \alpha^* + \mu^*_i + \eta^*_{it} \tag{A.3}$$

but
$$E(\eta^*_{it}\ \eta^*_{i\tau}) \neq 0, \quad t \neq \tau \quad .$$

We wish to show that the test of whether $E[\eta^*_{it}|\mu^*_i] = 0$ can be performed by testing whether $E[\eta_{it}|\mu_i] = 0$ in the transformed system (A.1).

Suppose
$$\eta^*_{it} = \phi_i(L)\eta^*_{it} + \eta_{it} \quad , \tag{A.4}$$

$$E[\eta_{it},\eta_{i\tau}] = 0, \quad t \neq \tau \tag{A.5}$$

Given (A.4) and (A.5), we can use the typical "quasi-first-differencing"procedure in (A.3) to obtain:

$$(ROE^*_{it} - \phi_i(L)ROE^*_{it-1}) = \alpha^*(1-\phi_i(L)) + \mu^*_i(1-\phi_i(L)) + (\eta^*_{it} - \phi_i(L)\eta^*_{it-1}) \tag{A.6}$$

$$\Rightarrow \qquad ROE_{it} = \alpha + \mu_i + \eta_{it} \tag{A.7}$$

which is (A.1).

Now, will the test $E[\eta^2_{it}|\mu_i] = 0$ in (A.7) provide a valid test of $E[\eta^*_{it}|\mu^*_i] = 0$ in (A.3)? Taking the AR(1) case for simplicity, and remembering that if $|\phi| < 1$, $\sigma^2_{\eta i} = (1 - \phi^2_i)\phi^2_{\eta * i}$,

$$E[\phi^2_{\eta i}|\mu_i] = E[(1-\phi^2_i)\sigma^2_{\eta *}|(1-\phi^*_i(L_i))\mu\ ]$$

$$\equiv E[g_1(\phi_i(L))\sigma^2_{\eta *}\ |\ \lambda_2(\phi_i(L))\mu^*_i]$$

$$= E_{\phi_i(L)}\{\ E[g_1(\phi_i(L))\sigma^2_{h*}\ |\ g_2(\phi_i(L))\mu^*_i\ |\ \phi_i(L)]\ \}$$

$$= E_{\phi_i(L)}\{g_1(\phi_i(L))E[\sigma^2_{\eta *}\ |\ g_2(\phi_i(L))\mu^*_i\ |\ \phi_i(L)]\ \} \tag{A.8}$$

APPENDIX B

For simplicity, define $r_{it} \equiv ROE_{it}$. Then the error components model in the text is:

$$ROE_{it} \equiv r_{it} = \alpha + \mu_i + \eta_{it} \tag{B.1}$$

Following the procedure in Marsh and Newey [1983], taking time averages in (B.1) for any i gives:

$$\bar{r}_{i.} = \alpha + \mu_i + \bar{\eta}_{i.} \tag{B.2}$$

where

$$\bar{r}_{i.} = \frac{1}{T} \sum_{t=1}^{T} r_{it} ,$$

$$\bar{\eta}_{i.} = \frac{1}{T} \sum_{t=1}^{T} \eta_{it} \tag{B.3}$$

$$\hat{\sigma}_i^2 = \frac{1}{T-1} \sum_{t=1}^{T} (r_{it} - \bar{r}_{it})^2 = \frac{1}{T-1} \sum_{t=1}^{T} (\eta_{it} - \bar{\eta}_{i.})^2$$

Under the null hypothesis H_o:

$$E_i \ (\eta_{it}^2 \mid \mu_i) = \sigma_\eta^2$$

$$E_i \ (\eta_{it} \mid \mu_i) = 0$$

$$E_i(\eta_{it}\eta_{is}^r \mid \mu_i) = 0 \qquad t \neq s, \ r > 0 \tag{B.4}$$

Under the alternative hypothesis,

$$E_i\ (\eta_i^2 \mid \mu_i) = g(\mu_i)$$

$$E_i\ (\eta_{it} \mid \mu_i) = 0 \tag{B.5}$$

$$E_i\ (\eta_{it}\eta_{is}^r \mid \mu_i) = 0 \qquad t \neq s,\ r > 0$$

The test is developed under the null hypothesis. From (B.2) and B.3):

$$E_i(\bar{r}_{i.}\ \hat{\sigma}_i^2) = E_i[\alpha + \mu_i + \bar{\eta}_{i.},\ \hat{\sigma}_i^2\ ] \tag{B.6}$$

$$= \alpha E_i(\hat{\sigma}_i^2) + E_i(\mu_i\hat{\sigma}_i^2) + E_i(\bar{\eta}_{i.}\hat{\sigma}_i^2) \tag{B.7}$$

As long as the <u>cross-sectional</u> number of firms, N, is large,

$$\lim_{N\to\infty}\ E_i(\hat{\sigma}_i^2) \to \sigma^2 = \sigma_\eta^2 \tag{B.8}$$

Further, under the null hypothesis (B.4):

$$E[\mu_i\hat{\sigma}_i^2] = E_{\mu i}[E(\mu_i\hat{\sigma}_i^2) \mid \mu_i] = E_{\mu i}[\mu_i E(\hat{\sigma}_i^2 \mid \mu_i)] = 0 \tag{B.9}$$

$$E_i\ [\bar{\eta}_{i.}\ \hat{\sigma}_i^2] = \frac{1}{T}\ E_i\ (\eta_{it}^3\ ) \equiv \frac{1}{T}\ \mu_3 \qquad \text{if N is large} \tag{B.10}$$

Thus,

$$E_i\ (\bar{r}_{i.}\hat{\sigma}_i^2) = \alpha\ \sigma_\eta^2 + \frac{\mu_3}{T}$$

$$E_i(\hat{\mu}_3) = E_i[\ \frac{T}{(T-1)(T-2)}\ \sum_{t=1}^{T}(r_{it} - \bar{r}_{i.})^3]$$

$$E_i(\bar{r}_{i.}) = \alpha$$

By an appropriate central limit theorem,

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \begin{bmatrix} \bar{r}_{i.}\hat{\sigma}_i^2 - \alpha\sigma_\eta^2 - \frac{\mu_3}{T} \\ \bar{r}_{i.} - \alpha \\ \hat{\sigma}_i^2 - \sigma_\eta^2 \\ \hat{\mu}_{3i} - \mu_3 \end{bmatrix} \xrightarrow{d} N(0,\tilde{\Omega})$$

where the asymptotic variance matrix $\tilde{\Omega}$ can be consistently estimated by:

$$\hat{L} = \frac{1}{N-1} = \sum_{i=1}^{N} \begin{bmatrix} (\bar{r}_{i.}\hat{\sigma}_{i.}^2)^2 & \bar{r}_{i.}^2\hat{\sigma}_{i.}^2 & \bar{r}_{i.}(\hat{\sigma}_{i.}^2)^2 & \bar{r}_{i.}\hat{\mu}_{3i.}\hat{\sigma}_{i.}^2 \\ & r_{i.}^{-2} & \bar{r}_{i.}\hat{s}_{i.}^2 & \bar{r}_{i.}\hat{m}_{3i.} \\ & & (\hat{\sigma}_{i.}^2)^2 & \hat{\sigma}_{i.}^2\hat{\mu}_{3i.} \\ & & & (\hat{\mu}_{3i.})^2 \end{bmatrix} \tag{B.11}$$

$$- \frac{1}{N} \begin{bmatrix} \sum_{i=1}^{N} \bar{r}_{i.}\hat{\sigma}_i^2 \\ \sum_{i=1}^{N} \bar{r}_{i.} \\ \sum_{i=1}^{N} \hat{\sigma}_{i.}^2 \\ \sum_{i=1}^{N} \hat{\mu}_{3i.} \end{bmatrix} [\sum_{i=1}^{N} \bar{r}_{i.}\hat{\sigma}_{i.}^2, \sum_{i=1}^{N} \bar{r}_{i.}, \sum_{i=1}^{N} \hat{\sigma}_{i.}^2, \sum_{i=1}^{N} \hat{\mu}_{3i.}] \}$$

Defining

$$q_N = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \bar{r}_{i.}\hat{\sigma}^2_{i.} - \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \bar{r}_{i.}\right)\left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \hat{\sigma}^2_{i.}\right) - \frac{1}{T}\left[\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \hat{\mu}_{3i.}\right] \tag{B.12}$$

then the asymptotic variance of q_N is consistently estimated by:

$$\hat{V}(q_N) = \left[1, \quad -\frac{1}{N}\sum_{i=1}^{N} \hat{\sigma}^2_{i.}, \quad -\frac{1}{N}\sum_{i=1}^{N} \bar{r}_{i.}, \quad -\frac{1}{T}\right] \hat{\Omega} \begin{bmatrix} 1 \\ -\frac{1}{N}\sum_{i=1}^{N} \hat{\sigma}^2_{i.} \\ -\frac{1}{N}\sum_{i=1}^{N} \bar{r}_{i.} \\ -\frac{1}{T} \end{bmatrix} \tag{B.13}$$

and

$$S = \frac{q_N^2}{\hat{V}(q_N)} \quad \dashrightarrow \quad \chi^2(1) \tag{B.14}$$

Table 1

Mean and median of autocorrelation coefficienta of ROE series for one, two, three, and four quarter lags and one year lag.

ROE Series

Autocorrelation Calculated for:	(EPS/MV)[1] 1958-1969	(EPS/MV)[1] 1970-1981	(EPS/MV)[1] 1958-1981	(EPS/MV)[1] 1963-1981	(EPS/BV)[2] 1964-1981	(EPS/BV(-1)[3] 1963-1981
One Quarter Lag						
Mean	.314	.451	.529			
Median	.330	.496	.569			
Two Quarter Lag						
Mean	.116	.350	.424			
Median	.161	.377	.480			
Three Quarter Lag						
Mean	.120	.296	.406			
Median	.102	.275	.405			
Four Quarter Lag						
Mean	.429	.422	.525			
Median	.408	.432	.564			
One-Year Lag (on annual data)						
Mean	.360	.504	.632	.615	.519	.518
Median	.392	.537	.684	.673	.560	.543

[1]ROE defined as quarterly primary earnings per share excluding extraordinary items divided by end of period market value of equity per share.

[2]ROE defined as annual primary earnings per share excluding extraordinary items divided by end of period book value per share (common equity divided by the number of sharea used to calculate primary earnings per share).

[3]ROE defined as annual primary earnings per share excluding extraordinary items divided by beginning of year book value per share (common equity divided by the number of shares used to calculate primary earnings per share).

Table 2

Table of industries studied including SIC code, industry name, and number of firms

SIC Code	# of firms	Name of Industry*
10	4	Metal Mining
20	10	Food and Kindred Products
22	6	Textile Mill Products
26	4	Paper and Allied Products
28	11	Chemicals and Alliled Products
29	12	Petroleum and Coal Products
32	7	Stone, Clay, and Glass Products
33	7	Primary Metal Industries
35	14	Machinery, except electrical
36	11	Electric and Electronic Equipment
37	11	Transportation Equipment
38	5	Instruments and Related Products
49	13	Electrical, Gas, and Sanitary Services

* Name based upon 9/21/78 SIC classification

Table 3

Rank Orders of Annual ROE[1] Variance vs. Rank Order of Annual ROE Means for period 1958-1981 of firms on an industry basis for all industries with 4 or more firms. Contingency table of sample ROE mean vs. sample ROE variance based on rank order.[2]

Two-Digit SIC[3] Industry Code	Rank Order of ROE Means[4] 1	2	3	4	5	6	7	8	9	10	11	12	13	14	Contingency Tables		ROE Variance High	Low
	Rank Order of ROE Variance																	
10	2	4	3	1											ROE	High	1	1
															Mean	Low	1	1
20	10	1	3	2	5	4	6	8	9	7					ROE	High	4	1
															Mean	Low	1	4
22	4	2	6	1	3	5									ROE	High	2	1
															Mean	Low	1	2
26	1	2	4	3											ROE	High	2	0
															Mean	Low	0	2
28	1	4	2	6	3	5	9	7	10	11	8				ROE	High	5	1/2
															Mean	Low	1/2	5
29	11	1	3	4	5	7	2	6	10	8	9	12			ROE	High	4	2
															Mean	Low	2	4
32	1	6	5	2	3	7	4								ROE	High	1-1/2	2
															Mean	Low	2	1-1/2
33	7	3	1	6	2	4	5								ROE	High	2	1-1/2
															Mean	Low	1-1/2	2
35	4	1	3	14	12	10	2	9	7	6	5	11	8	13	ROE	High	4	3
															Mean	Low	3	4
36	11	10	1	2	7	4	8	3	5	6	9				ROE	High	2-1/2	3
															Mean	Low	3	2-1/2

(Table 3 - Continued)

Two-Digit SIC[3] Industry Code	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Contingency Tables		ROE Variance High	ROE Variance Low
	Rank Order of ROE Means[4] / Rank Order of ROE Variances																	
37	10	11	3	9	8	2	6	1	5	7	4				ROE	High	1-1/2	4
															Mean	Low	4	1-1/2
38	1	3	4	2	5										ROE	High	1-1/2	1
															Mean	Low	1	1-1/2
49	12	3	10	2	4	5	8	9	1	11	7	6	13		ROE	High	4	2-1/2
															Mean	Low	2-1/2	4

[1]ROE defined as annual primary earnings per share excluding extraordinary items divided by end of period market value of equity per share.

[2]Two-digit SIC Code as obtained from the 1978 Quarterly Industrial COMPUSTAT file dated September 21, 1978.

[3]High and low ROE mean and variance as compared to the median. Median counted as half in high and half in low.

[4]For each industry, the entries in the corresponding row are ranks of the sample ROE variances for the firms within the industry (the maximum number of firms in any industry is 14).

Table 4

Rank Ordera of Annual ROE[1] Variance vs. Rank Order of Annual ROE Means for period 1964-1981 of firms on an industry basis for all industries with 4 or more firms. Contingency table of sample ROE mean vs. sample ROE variance based on rank order.[2]

Two-Digit SIC[3] Industry Code	Rank Order of ROE Means[4] 1	2	3	4	5	6	7	8	9	10	11	12	13	14	Contingency Tables		ROE Variance High	ROE Variance Low
	Rank Order of ROE Variances																	
10	3	1	2	4											ROE	High	1	1
															Mean	Low	1	1
20	9	7	10	3	4	6	8	2	1	5					ROE	High	2	3
															Mean	Low	3	2
22	4	6	1	2	5	3									ROE	High	1	2
															Mean	Low	2	1
26	4	2	1	3											ROE	High	1	1
															Mean	Low	1	1
28	9	8	7	1	6	2	5	11	4	10	3				ROE	High	2	3-1/2
															Mean	Low	3-1/2	2
29	11	2	1	3	8	7	6	10	5	9	4	12			ROE	High	3	3
															Mean	Low	3	3
32	7	3	6	4	2	5	1								ROE	High	1-1/4	2-1/4
															Mean	Low	2-1/4	1-1/4
33	7	1	2	4	3	5	6								ROE	High	2-1/4	1-1/4
															Mean	Low	1-1/4	2-1/4
35	12	9	13	10	2	5	14	6	7	3	8	4	11	1	ROE	High	2	5
															Mean	Low	5	2
36	10	11	1	3	2	5	4	8	9	7	6				ROE	High	3-1/2	2
															Mean	Low	2	3-1/2

(Table 4 - Continued)

Two-Digit SIC[3] Industry Code	Rank Order of ROE Means[4] 1	2	3	4	5	6	7	8	9	10	11	12	13	14	Contingency Tables		ROE Variance High	Low
	Rank Order of ROE Variance																	
37	10	1	3	2	7	6	5	8	4	9	11				ROE	High	3-1/4	2-1/4
															Mean	Low	2-1/4	3-1/4
38	2	4	3	1	5										ROE	High	1-1/4	1-1/4
															Mean	Low	1-1/4	1-1/4
49	12	10	5	3	4	7	6	8	9	2	11	1	13		ROE	High	4	2-1/2
															Mean	Low	2-1/2	4

[1]ROE defined as annual primary earnings per ahare excluding extraordinary items divided by beginning of year book value per share (common equity divided by shares used to calculate primary esrnings per share).

[2]High and low ROE mean and variance determined relative to the median.

[3]Two-digit SIC code as obtained from the 1978 COMPUSTAT Quarterly Industrial file dated September 21, 1978.

[4]For each industry, the entries in the corresponding row are ranks of the sample mean ROEs for the firms within the industry (the maximum number of firms in any industry is 14).

Table 5

Intro-industry correlation between ROE mean and ROE variance for different time periods based upon 2x2 contingency tables and Spearman tests on rank orders of ROE mean and ROE variance.

Type of Series	# of industries with positive correlation in contingency Table[1]	# of industries with negative correlation in contingency Table[1]	# of industries with positive correlation using Spearman test[2]	# of industries with negative correlation using Spearman test[2]
#1-EPS/BV 1963-1981 Annual	5	7	4	9
#2-EPS/BV(-1) 1964-1981 Annual	4	5	5	7
#3-EPS/MV 1963-1981 Annual	9 (1)	4	9	4
#5-EPS/MV 1958-1981 Annual	9 (1)	3	9	4
#5-EPS/MV 1958-1969 Annual	8	4	9	4
#5-EPS/MV 1970-1981 Annual	9 (1)	3	9	4
#4-EPS/MV 1958-1981 Quarterly	7 (1)	4	10	3
#4-EPS/MV 1958-1969 Quarterly	9	3	11	2
#4-EPS/MV 1970-1981 Quarterly	9 (1)	3	9	4
#6-EPS/MV(+1) 1958-1980 Annual	10 (1)	2	9	4
#6-EPS/MV(+1) 1958-1969 Annual	9	4	9	4
#6-EPS/MV(+1) 1970-1980 Annual	10 (1)	1	10	3
#6-EPS/MV(+1) 1958-1981 Q'ly	8 (1)	3	10	3
#6-EPS/MV(+1) 1958-1969 Q'ly	10	2	10	3
#6-EPS/MV(+1) 1970-1981 Q'ly	8 (1)	3	9	4

[1]2x2 contingency tables based upon rank order of sample ROE Mean and sample ROE variance. Number of significant observations at the 95% confidence level based upon a chi-square test is in parenthesis.

[2]Spearman test performed by calculating the correlation coefficient between rank orders of sample ROE mean and sample ROE variance.

Table 6

Correlation for whole sample of 135 firms between sample ROE means and sample ROE variances based upon 2x2 contingency tables and Spearman tests for the different ROE definitions and time periods indicated.

Type of Series	Correlation and chi-square statistic of 2x2 contingency table of ROE mean and ROE variance[1]		Spearman test correlation coefficient[2]
#1-EPS/BV 1963-1981 Annual	(-)	.36	-.196
#2-EPS/BV(-1) 1964-1981 Annual	(-)	.01	-.081
#3-EPS/MV 1963-1981 Annual	(+)	13.70	.324
#5-EPS/MV 1958-1981 Annual	(+)	15.00	.329
#5-EPS/MV 1958-1969 Annual	(+)	15.00	.395
#5-EPS/MV 1970-1981 Annual	(+)	12.45	.276
#4-EPS/MV 1958-1981 Quarterly	(+)	15.00	.306
#4-EPS/MV 1958-1969 Quarterly	(+)	19.27	.447
#4-EPS/MV 1970-1981 Quarterly	(+)	12.45	.285
#6-EPS/MV(+1) 1958-1980 Annual	(+)	15.00	.364
#6-EPS/MV(+1) 1958-1969 Annual	(+)	9.07	.410
#6-EPS/MV(+1) 1970-1980 Annual	(+)	12.45	.326
#6-EPS/MV(+1) 1958-1981 Q'ly	(+)	15.00	.344
#6-EPS/MV(+1) 1958-1969 Q'ly	(+)	17.79	.444
#6-EPS/MV(+1) 1970-1981 Q'ly	(+)	19.27	.346

[1]Direction of correlation, either positive (+) the or negative (-), in parenthesis.

[2]The Spearman test is performed by calculating the correlation coefficient between rank orders of sample ROE mean and sample ROE variance.

Table 7

Intra-industry and whole sample correlation and test statistics from the error components test for cross-sectional association between ROE means and ROE variances using four different definitions of ROE.[2]

Industry SIC code 2	# of firms in industry	Correlation and test statistic for ROE Series 1 EPS/BV 1963-1980		Correlation and test statistic for ROE series 2 EPS/BV(-1)1965-81		Correlation and test statistic for ROE series 4 EPS/MV 1958-1981		Correlation and test statistic for ROE series 6 EPS/RV(+1)1958-81	
10	4	(+)	2.12	(+)	0.30	(-)	0.04	(+)	0.54
20	10	(-)	2.23	(+)	0.67	(-)	0.00	(-)	0.25
22	6	(+)	10.89	(-)	1.14	(-)	3.76	(-)	2.33
26	4	(+)	0.00	(-)	0.02	(+)	0.12	(-)	0.01
28	11	(+)	0.31	(+)	0.00	(+)	3.12	(-)	0.06
29	12	(-)	1.96	(-)	1.61	(-)	0.71	(-)	0.62
32	7	(-)	0.60	(-)	0.97	(-)	4.46	(-)	0.28
33	7	(+)	0.66	(+)	0.03	(+)	0.37	(+)	0.24
35	14	(+)	5.65	(+)	4.69	(+)	1.22	(+)	2.91
36	11	(+)	0.52	(-)	0.00	(+)	0.76	(+)	2.44
37	11	(-)	0.39	(-)	0.53	(+)	1.10	(+)	1.13
38	5	(-)	0.09	(-)	0.14	(+)	0.23	(+)	1.23
49	13	(+)	0.71	(+)	0.20	(-)	0.01	(+)	1.71
Market	135	(+)	1.09	(+)	1.93	(+)	3.04	(+)	1.25

[1]Direction of correlation, either positive (+) or negative (-), in parenthesis.

[2]2-digit SIC code, obtained from the 1978 Quarterly Industrial COMPUSTAT file dated September 21, 1978.

Figure 1

Histogram of lag one one-quarter autocorrelations of ROE Series 4 (EPS/End-of-quarter market value of equity for the period 1958-1981) by decile from -1.0 to 1.0. Total number of firms equals 135.

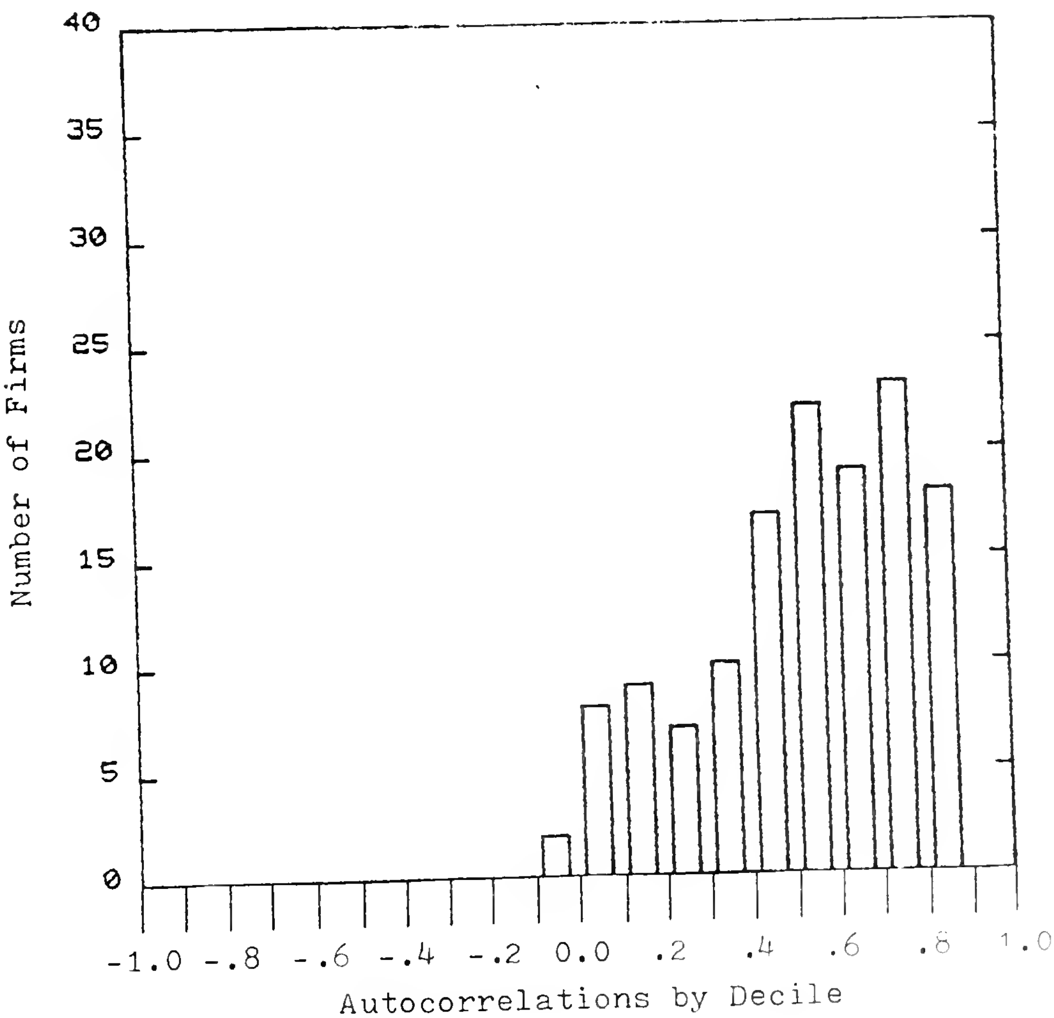


Figure 2

Histogram of lag one one-year autocorrelations of ROE Series 5 (EPS/End-of-year market value of equity for the period 1958-1981) by decile from -1.0 to 1.0. Total number of firms equals 135.

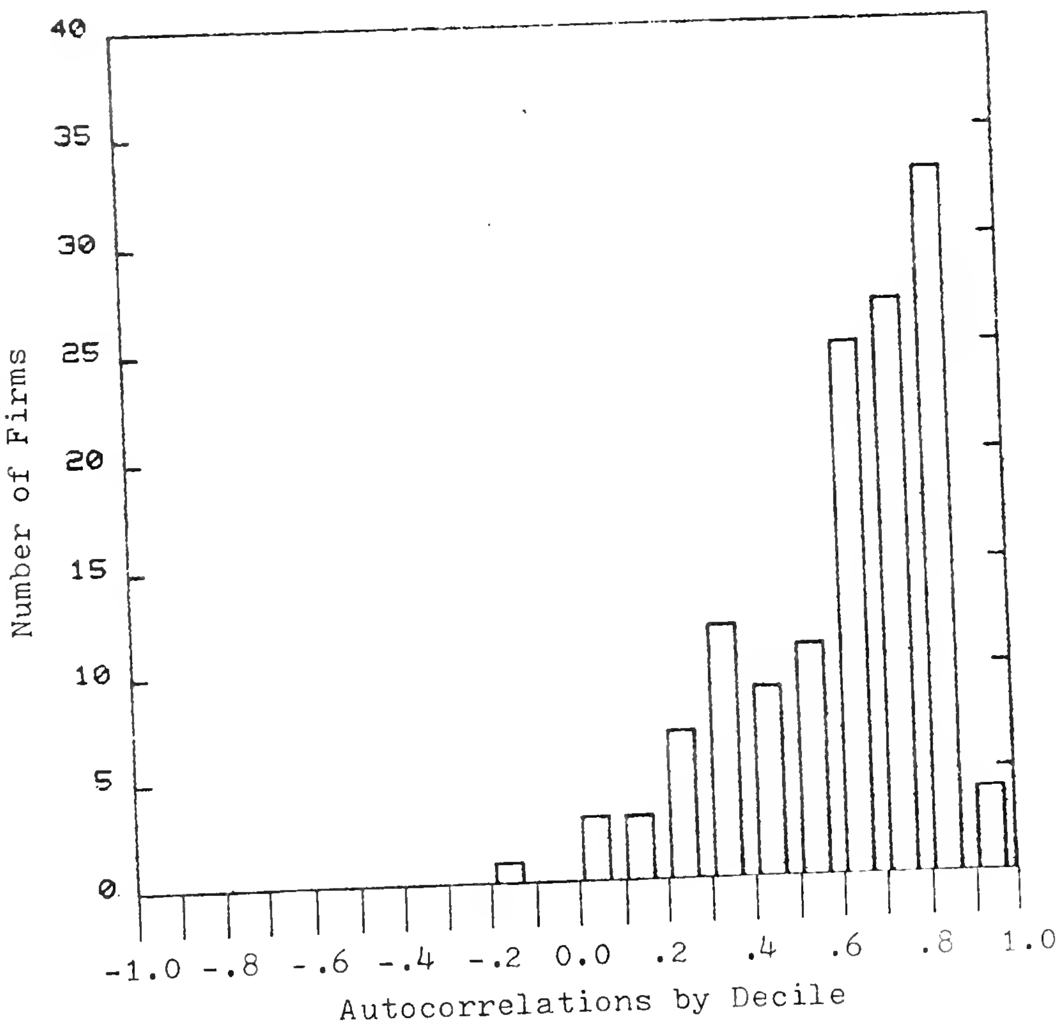
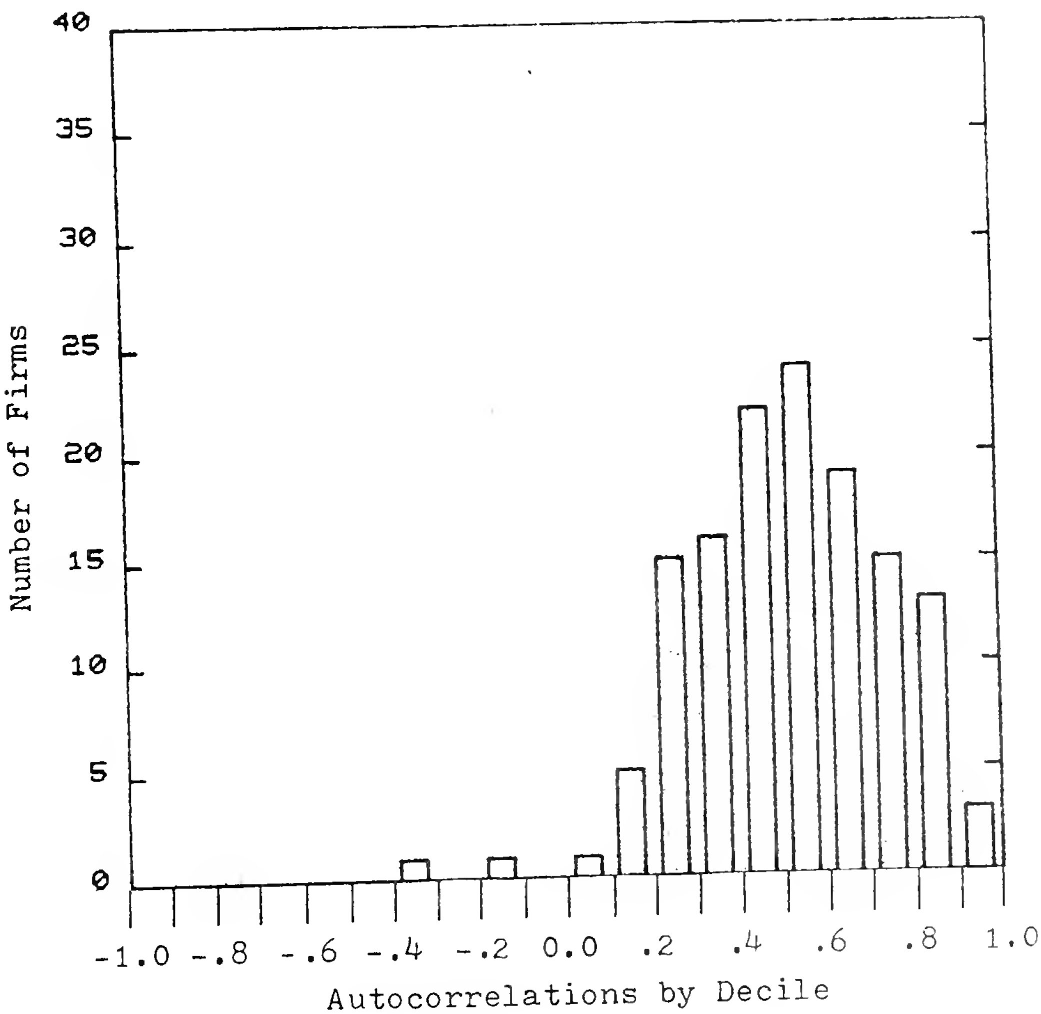


Figure 3

Histogram of lag one one-year autocorrelations of ROE Series 2 (EPS/Beginning-of-year book value of equity for the period 1964-1981) by decile from -1.0 to 1.0. Total number of firms equals 135.



8

0234 025